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Swarm_Intelligence (SI)-State-of-Art (SI-SA) Part 1[#]: Tutorial on Firefly algorithm

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(Dedicated with reverence to Prof G Kateman, core Chemometrician, University of Nijmegan, Netherlands, on his eightieth-birth anniversary)

ABSTRACT

In fireflies of Lampyridae family, bioluminescence with glowing and flashing abdomens is a communication signal for courtship. The photo-chemical processes in these natural fireflies are also to find and absorb preys and protect themselves from predators. Yang et al. put forward (artificial) firefly algorithm, another member of metaheuristic bandwagon. The attractiveness between fireflies is based on light intensity which in turn depends upon the floating point values of object function. The less bright-fireflies flock around and forms a neighbourhood around brighter ones. The algorithm operating in iterative improvement of solutions enables locating simultaneous multiple local and global extrema (minima/maxima) of non-linear non-convex multi-modal mixed-variable and constrained functions with breaks and singularity in the search space. The impetus for movement of fireflies lies in attraction component locating minima and random factor to drive away from trapping in local optima. Firefly algorithm behaves like a scaled random method or part of PSO (particle swarm optimisation) depending upon setting of free parameters. The applications spread their wings into chemistry, medicine, chemical technology, clustering, engineering etc. The loading pattern of fuel assemblies in PWR (pressurised water reactor in atomic energy), phase equilibria, separation of DNA/RNA, medical images in cancer research, truss structures, electrical power generation/distribution are a few of typical conflicting multi-objective hard tasks solved with firefly algorithm. The modifications widening functional capabilities of firefly algorithm include replacing random component by chaotic maps and hybridisation with genetic algorithm (GA), differential evolution (Diff.Evol), mimetic approach, eagle, simulating annealing, ant colony, learning automata and NNs.

Keywords: Glow-worm, Lévy flights, Multi-object-functions, E-man, Nature mimicking, Chemical engineering, cancer diagnosis.

Contents_ Firefly algorithm

INTRODUCTION

Bio inspiration is a splinter in the fire of nature. Chemistry, physics, biology, geology, astronomy etc. are sparks of splinters from the moment of origin of universe till to date. The automated pattern recognition, systematic fault detection, machine learning and adaptive optimisation are software procedures. In spite of incomplete understanding, shortcoming in translation, limitations of implementation etc. these tools are far superior to even state-of-the-art-of-mathematical-statistical techniques. Robots in defense, disaster management, surgery, environment, space/underwater exploration, entertainment and thinking are combinations of hardware/firmware with inspiration from best-/worst-/taboo- processes of insects/animals/human beings/even imaginary species of intuition mapping. In recent years, Yang put forward firefly [1], cuckoo [2] and bat [3,4] algorithms and applied in computer science and engineering. The digestion, locomotion and reproduction are the basic processes operated in biological life. With evolution, added natural instincts are foraging, retention of their genes, home building, defense against predators, wild approach for occupation of territory/female clad, courting, grouping/swarming, adapting to harsh-environment/poisonous-predators and so on. The context based communication through sound, light, dance etc. is also diverse, and unique in its own sense in many species. Corridor walk of cat, rat moving around, pheromone trails of ants, waggle dance of honey bees, herd of wild animals (lions, owls, dolphins) hunting their prey, speed/ concentration strategies of bat are long lived astounding foraging procedures. In continuation of our efforts in chemometrics [5,6] piscimetrics [7] neural networks [8-10] during the quarter century and Eman [11,12] recently, the current status of firefly algorithm and their applications are presented in this communication. A search from Scopus resulted in around 170 abstracts and full papers from electronic journals also are the primary source of information.

1.1 Fireflies in nature: Fire flies, also called glowworms or lightning bugs are found all over the world from North/South America to Europe. And they are also present in Asia. There are about two thousand species of fireflies exhibiting bioluminescence. Fireflies are beetles, members of the family Lampyridae. They flash [13,14] in pale yellow to reddish green color (in the range between 510 and 670 nanometers) attractively in dark in meadows with varying brightness in the summer sky in the tropical temperature regions. The emission of luciferin (Appendix 1) is the cause of this bioluminescence. A tube in the abdomen called the abdominal trachea supplies oxygen to the cells. These cells also contain uric acid promoting reflection of the light away from the abdomen. This bliss of nature (of on-off switching of the light) in fireflies is a fantastic communicating signal system inducing an intelligent social behavior. Fireflies with this trait of synchronized flashing find and absorb preys, protect themselves from predators and attract other fireflies to their vicinities for mating. The habitat of the fireflies is usually humid vegetation terrine land. These environments protect them from desiccation, provide source of food/ hiding place/ space for roaming and a safe zone to lay eggs. The attraction between any two fireflies is proportional to their brightness. But, the environment (even air) absorbs light and thus flashes of fireflies fades away beyond a few hundreds of meters from a firefly (source). Obviously, the natural trait is that less bright ones are attracted and move towards a brighter one, sometimes even flock around it. Interestingly, the brightest one moves in a random fashion, as no other firefly can attract it. Female fireflies of a category of species send wrong signals to attract males of another category making them believe it is for courting. When the male approaches, female eats the male fly. Chart 1 abridges how swarm of tiny unintelligent life systems performs stupendous tasks through distinct communication means.

Chart 1: Communication and goals in typical (natural) swarm of species

Nature	Birds	Ants	Honey bees	Firefly
Communication	■ Electro magnetic signals	Ⓛ Laying of pheromones	➔ Waggle dance	ॐ Bioluminescence
Goal	■ Visiting a place never seen	Ⓛ Shortest path between ant hill and food source	➔ Flower patches of high quantity and quality nector	ॐ Prey detection ॐ Mating signal ॐ Signaling the presence of invaders

1.2. Translation of (natural) firefly bioluminescent attraction into multi-agent (swarm-intelligent) nature mimicking algorithm : The social behavior of fireflies to achieve basic needs of life is also a natural optimization feat in time and energy space through generations of species. This is similar to searching for the global optimum solution of a task. The information exchange and communication through bioluminescent light serves as a means of attracting other fireflies and preys. The information regarding object function value and direction of movement of profile during iterations and success/failure count (in terms of consecutive increasing or decreasing trends) is a navigator towards searching to global extrema. The natural traits, which are difficult to be mimicked in a laboratory or in software/integrated-chip have been referred as accepted intelligent characteristics of the species. Thus, the exciting or apparently strange behavior is an instance of referential collective intelligence. Here, the behavior of swarm of fireflies/glow-worms is translated to finding solution of an optimisation task (Chart 2).

Chart 2: Translation of nature (firefly)'s processes into Metaheuristic (firfly) algorithm

Fireflies	
Natural	**Artificial**
Lampyridae fireflies	Agents

[males, females]	unisex
Bioluminescence	Object function value
↓Decreases with ■ distance ■ environment	Euclidean distance
Attraction between flies	Eqn. 2
Movement	Eqn. 4
♥ Mating ♥ Eating prey	Convergence to minimum

Chart 2(b): Terminology in typical nature inspired swarm algorithms

Algorithm				
Mathematical	**Birds**	**Ants**	**Honey bees**	**Firefly**
x	■ Particle	Ants	Honey bees	ॐ Firefly
xapproximate	■ Positon ■ velocity	Pheromone quantity	position	ॐ Position
Function	■		Mass	ॐ Attraction
Number of app solutions	■ Number of birds	Number of ants	Number of honey bees	ॐ Number of fireflies

2. (Artificial) firefly algorithm: Yang [1] put forward firefly algorithm with inspiration from collective social behavior of fireflies promoted by communication through bioluminescence of characteristic different glittering patterns of flashes. It is another population based metaheuristic algorithm and used [15-184] in nonlinear multimodal optimization in dynamic environment. The artificial fireflies (agents) move towards brighter points to land on a global optimum point in the search space. Fireflies in nature and also in algorithm are swarm (collective behavior of more than two) based. Swarm behavior is an unorganized/disorganized/not organized cluster of species moving irregularly/chaotically in space and time without a leader.

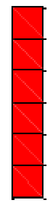
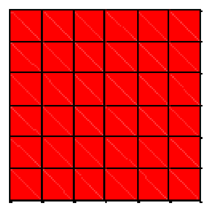
2.1 Assumptions_firefly algorithm

Sex of fireflies: The fireflies are unisexual. Thus, each firefly attracts the rest. However, if all the fireflies have the same sex, no mutation operation is possible to expect positive surprising moves. But, mutation operator has the advantage of altering the attraction between fireflies to escape from local optima and also to increase chances of trending towards global extremum. To circumvent this shortcoming, fireflies are two gender type flies in some algorithms. This promotes the movement of flies towards optimum more in right direction exploiting reasonable mutation on combination.

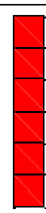
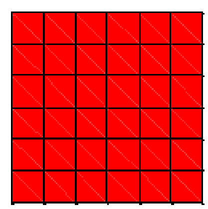
2.2 Data_structure_firefly algorithm : The primary data, derived variables and algorithm specific fixed constants/variables are incorporated in Chart 3. The programs are developed in this laboratory as matlab functions coded in MATLAB R2011 and all test runs are performed on a Dell laptop with Intel Core(TM) i7-2670QM CPU @ Processor (2.23 GHz) and 8GB of RAM under Windows 7 Ultimate (32-bit) operating system. Word from Office Professional 2010 is employed for word-processing and publishing tasks.

Chart 3: Data structure of Firefly algorithm

(a) Variables and fixed parameters

Variable	Column Vector	Abbreviations and full form
$objFnvalue : \begin{bmatrix} y_1 \\ y_2 \\ y_i \\ y_{nsol} \end{bmatrix}$	 **y**(nsol,1) change color	
$position : \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,d} \\ x_{2,1} & x_{2,2} & x_{2,d} \\ x_{i,1} & x_{i,2} & x_{i,d} \\ x_{nsol,1} & x_{nsol,2} & x_{nsol,d} \end{bmatrix}$	**MATRIX**  **X**(nsol,dim)	

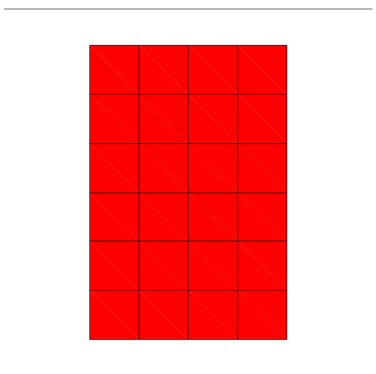
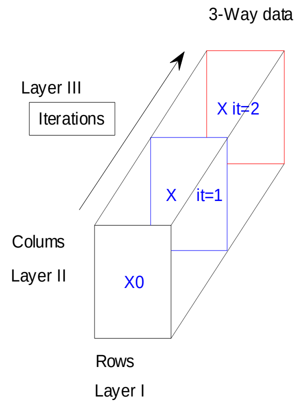
Nsol	:	Number of solutions (fireflies/ agents)
y_i	:	Objective function (or fitness) value of *ith* agent (firefly) (e.g. ESS)
x_{id}	:	Coordinates of i[th] firefly's position in d[th] dimension

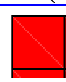
	Size of tensor		
	[nsol x 1]	[nsol x iter]	
variable_firefly alg			
	1 Column vector	2	Order of tensor
	Initial	iterations	
y(objFnValue)	y0	y_iter	

Scaling factor:

If	Range of X in ith dimension $\left(\left[10^{-5}\ to\ 10^{+5}\right]\right)$ widely differs from Jth dimension $\left(\left[-0.001\ to\ 0.01\right]\right)$
Then	Use scaling factor $\left(scale\_factorX\right)$ [169]

Initial	Iterations
Second order tensor (Matrix):	**Third order tensor (3-way data):**

[nsol x ndim]	[nsol x ndim x iter]
	
X0	Xiter
dist	dist_iter
Attract_f1f2	Attract_f1f2_iter
nonDeterministic	nonDeterministic_iter
delX	delX_iter
X	X_iter

Fixed parameters						
General optimization			zero order tensor (scalar) : 1 x 1	Firefly specific		
Maximum Iterations	:	maxIter		Reduction of randomisation	:	rand_reduce, (α)
				Absorption coefficient	:	absorbCcoef, (γ)
				Coefficient of attraction	:	attractCoef

2.3 Equations and pseudo code of firefly metaheuristic : The equations and solution methods mostly in tensor format and typical matlab functions with simple numerical data follow.

2.3.1 Intensity of glow of fireflies: The intensity of glow of agents (i.e. brightness of artificial fireflies) is proportional to reciprocal of object function value at that instant (Chart 4).

Chart 4: KB for intensity and object function

If Goal is maximization of SOO

Then $Intensity\_light \ \alpha \ ObjFnValue$

If Goal is minimization of SOO

Then $$Intensity\_light \ \alpha \ \frac{1}{ObjFnValue} \quad \text{Eqn. 1}$$

If	Proportionality constant is unity & maximization
Then	$Intensity\_light = ObjFnValue$

2.3.2 Attraction between any two fireflies: The intensity of light emitted by a firefly is a measure of its attractiveness (Appendix-1). In other words, it is ironically expressed in terms of light intensity as seen by other fireflies. The attractiveness between two fireflies at zero distance is defined as attract_ fifj0 (β_0). Typical values of attract_fifj0 used are 0.1 to 10.0 (Table 1). Gandomi et al. [57] used chaotic maps normalized between 0 and 2 for attractCoef and found a significant improvement in the performance of firefly algorithm (Chart 5).

Influence of inter-distance between fireflies on attraction: The sharing of information between artificial fireflies (agents) is a function of their inter distance. The quantization is modeled as decrease of attraction with increase in square of Cartesian or Euclidean distance.

Chart 5: **Quantification of attraction between any two fireflies**

$$attract\_xixj = attractCoef0 * \exp^{-absorbCoef*(dist)^{power}}$$

Eqn.2

attractCoef0 (β_0)	:	Atttractiveness at dist=0
power	:	>0
dist	:	[Euclidean Hamming Manhattan …]

input				output
attractCoef= 0.2 attract_0 = 0.2 power = 2		om_attract.m		attractX = 0.16375 4.1223e-10 0
distX = 1 10 100				

```
%
% om_attract.m    R S Rao 19-10-13
%
function [attract_fifj,distXIter] =
om_attract(attractCoef0,absorbcoe,distX,power,iter)
attract= attractCoef0 * exp(-absorbCoef * dist .^power)
attract_fifj(:,:,iter) = attract
distXIter(:,:,iter) = distX;
```

2.3.3 Non-deterministic contribution for the movement fireflies: The simplest random component in Δx employed in movement of fireflies is uniform random number (Chart 6).

Chart 6: Calculation of non-deterministic component

```
%
%    om_rand.m
%
function [randX, nondeterministic] =  om_rand(distX,iter)
[row,col] = size(distX)
randX = rand(row,col)
nondeterministic(:,:,iter) = randX
```

Randu	:	Uniform random number
(1,1)	:	One number (scalar)

Randomness reduction (rand_reduce, α): Sometimes, oscillatory trend occurs in the solutions. A randomization reduction scheme surmounts this casualty (Chart 7). Thus, instead of using the randomness from probabilistic distribution (uniform, random etc.) as it is, reduction or toning down of randomness to some extent through rand_reduce coefficient is desirable.

Chart 7 : Typical formulae for computation of randomness reduction/tone_down

$$delta1 = \frac{(0.9-1.0e-4)}{10^{\left(\frac{1}{iterMax}\right)}} \quad check$$

$$randReduce.h1 = rand\_reduce * delta1$$

If Randomness is reduced fast
Then Premature convergence

Caution !! Use carefully

$$delta2 = (randReduce\_final - randReduce\_initial) * \frac{iter}{iterMax}$$

$$randReduce.h2 = randReduce\_initial + delta2$$

$$0 < reduction\_factor < 1$$

$$delta3 = (reduction\_factor)^{iter}$$

$$randReduce.h3 = delta3 * randReduce\_\mathrm{initial}$$

$$delta4(\mathrm{dimX}) = (scale\_factorX)$$

$$randReduce.h4 = randReduce\_initial * delta4$$

```
%
%om_randReduce.m
%
function [randReduce] =
om_randReduce(iter,iterMax,randReduce_initial,randReduce_final)
delta1 = (0.9-1e-4)/(10.^(1/iterMax))
randReduce.h1 = randReduce_initial *delta1
%
delta2 =  (randReduce_final - randReduce_initial) *
(iter/iterMax)
randReduce.h1 = randReduce_initial *delta2
```

```
%
delta3 =  (reduction_factor.^iter
randReduce.h1 = randReduce_initial *delta4
%
delta4 =  scale_factorX;
randReduce2.h1 = randReduce_initial *delta4
```

Levy flight: Also, Gaussian, Levy flight distribution (Chart 8) produced acceptable results. In general, the chaotic variable has special characteristics, i.e., ergodicity, pseudo-randomness and irregularity. The track of chaotic variable travels ergodically over the whole search space. An analogy is that the solution is drawn like a moth to a flame and cannot keep away. Although, other statistical distributions are valid and contemplated, they await detailed research.

Chart 8: Formulae for Levy flight profile

$1 < landa \le 3;$
$Levy1 = time^{(-landa)}$
	`c= 10`
	`for landa = c:c`
	`    Levy1(:,landa) = time.^(-landa);`
	`end`
	
	` Levy.h1 = Levy1;`
	` [time, Levy.h1]`
	` disp('***************')`
	`plot(time,Levy1,time,Levy1,'o')`
	`title('time.^(-landa)'`

$randsign = sign\left(rand(1,1) - \frac{1}{2}\right)$
$Levy2 = randsign * Levy1$
	`%`
	`%`
	`%`
	
	`[row,col] = size(Levy1);`
	`randsign = sign(rand(row,col)-0.5)`
	`Levy2 = randsign.* Levy1;`
	` Levy.h2 = Levy2;`
	` [time, Levy.h1]`
	` disp('***************')`
	`figure, plot(time,Levy2,time,Levy2,'o')`
	`title('Levy3 --Randsign')`

$\exp onent = \frac{3}{2}$
$A = \exp onent * gamaFn(\exp onent) * \frac{\sin\left(\frac{pi * \exp onent}{2}\right)}{pi}$
$Levy\,3 = A^{(-1-\exp onent)}$
	`%`
	`%`
	`%    check--- drop`
	`exponent = 3/2;`
	`exponent = time`
	`gammaX = gamma(exponent)`
	
	`A = exponent.* gammaX.* sin(pi*exponent/2)/pi`
	`Levy3 = A.^(-1-exponent)`
	`Levy.h3 = Levy3;`
	`%[time, Levy.h3]`
	` disp('***************')`
	`figure, plot(time,Levy3,time,Levy3,'o')`
	`title('Levy3')`

2.3.4 Movement (Δx) of *position* of fireflies due to attraction

Chart 9 : Calculation and heuristics for movement (Δx)of (artificial) fireflies	
If	ith firefly perceives more chemi-luminescence with jth firefly
Then	i^{th} firefly moves Δx distance towards j^{th} firefly
If	i^{th} firefly does not see any brighter fireflies
Then	i^{th} firefly moves randomly

The (artificial) fireflies (agents) change their position (Chart 9) in the (optimization function) search space (first term of Eqn. 3) depending upon the attractive forces. A random movement (second term of Eqn. 4) is coupled to run the iteration process to derive advantages of stochastic process. Yang proposed levy [148] distribution (Chart 8) for the second term instead of simple normal distribution.

Chart 9(b): components of Δx

$$\begin{aligned} & movement\_of\_ithfirefly \\ &\quad = mutual\,attraction + random\_\mathrm{term} \quad \mathrm{or} \\ &\quad = Intensification \quad + diversification \quad or \\ &\quad = Local\ search \quad + Global\ search \end{aligned}$$

Eqn.3

$$\Delta x(i, \mathrm{j}, iter) = attract\_\mathrm{fifj} * \left[\mathrm{x}(i, \mathrm{j}, iter) - \mathrm{x}(\mathrm{j}, \mathrm{j}, iter)\right] + rand\,\mathrm{Re}duce * rand$$

Eqn.4

$$\mathrm{x}(i, \mathrm{j}, iter+1) = \mathrm{x}(\mathrm{j}, \mathrm{j}, iter) + \Delta x(i, \mathrm{j}, iter)$$ Eqn.5

$$\mathrm{xI}(i, \mathrm{j}, iter+1) = \mathrm{int}\left(\mathrm{x}(i, \mathrm{j}, iter+1)\right)$$ Eqn.6

i^{th}	:	firefly
j^{th}	:	Dimension of X
iter	:	Iteration number
XI	:	Integer value of X

```
%
%    om_delX.m
%
function [delX] =  om_delX(attract_fifj,randReduce,nondeterministic,iter);
delX = attract_fifj + randReduce * nondeterministic
delXIter(:,:,iter) = delX

%
%om_refine.m
%
function [XIter,XOO] =  om_refine(X(iter),delX)
Xral = X + delX
XIter(:,:,iter) = Xreal
XOO.real = Xreal
XOO.integer = int(Xreal)
XOO.real = round (Xreal)
```

Chart 9(c) : Numerical expert system of movement operator in firefly algorithm

If $attractCoef0 = 0$

Then $attract\_fifj = 0$

$\Delta x(i, j, iter) = rand\,\mathrm{Re}duce * rand$

i.e. Pure (or non-cooperative distributed) random walk with a scale factor

If $attractCoef0 = 1$

Then $attract\_fifj = \exp^{-absorbCoef*(dist)^{power}}$

$$\Delta x(i, j, iter) = attract\_fifj * [x(i, j, iter) - x(j, j, iter)] + rand\,\mathrm{Re}duce * rand = cooperative\ search$$

i.e. Brightest firefly determines the position of other flies in the neighborhood

$$attract\_fifj = attractCoef0 * \exp^{-absorbCoef*(dist)^{power}} \quad \text{Eqn. 2}$$

2.3.5 Pseudo_code_firefly_algorithm : The algorithm of firefly in pseudo code form is depicted in Chart 10.

Chart 10: Pseudo-code for firefly algorithm Simple As possible (SAP)

Initialisation

Iterate

optimization

End Iteration

Output

Chart 10 (a) Initializations

Initialisation

Constants

Iteration dependent

Fire fly specific

Approximate solutions

If Initial/guess/approximate solutions are diverse

Then Converged solution is independent of initial population & settles to global solution

Chart 10 (b): Iteration in firefly algorithm	
Initialisation	
Iterate	
	cal light intensity (function of objFn) for a firefly considering effect of distance from its location
	cal pair wise mutual attraction of fireflies
	Cal objFnValue
	Move operator move the less bright firefly to the brighter one
	Updating firefly position
	Restore X if out of fence
end	

Chart 10 (c): Pseudo-code for firefly algorithm

```
Initialisation

Iterate  until convergence |
              stopping criteria
%%
%%  ------------ begin firefly_alg --------------------------
%%
   For  i=1:n (all n fireflies);
             for j=1:n (dim)

                 if       (Ij > Ii)
                 Then     move firefly i towards j
                 end if

                 Correct attractiveness for distance between fireflies
                 Cal current solutions
                 update light intensity

                 If       Constraint_for-X

                 Then     Constraint_Check

           endForj

   endFori

   %%  ~~~~~~~~~~~  firefly_alg  ends  ~~~~~~~~~~~~~~~~~

   sort fireflies
   cal current global best (glob_best)

       if       SAA
       Then     find the current global best
       end if
endIterate

Post-processing  results and visualization

Output
```

Chart 10 (d): Nondeterministic component of x

Current (iter) solution

Cal attraction ($attract\_xixj$) between
ith and jth fireflies
Cal non-deterministic component

if	deterministic
Then	$random\_term = 0$
if	Stochastic
Then	cal $random\_term$
if	Levy flights
Then	Cal $Levy3, Levy4$

$$\Delta x(i, j, iter) = deterministic + non\_deterministic$$

Chart 10 (e): Restoring out-of-bound X

if	Constrainted objFn
Then	Check_X_for_constraints
if	Constraint violation
Then	Set X to within constraints

Software: The code for firefly algorithm was developed in FORTRAN-90 [126], Matlab and Visual C#2008.

2.4 Iterative refinement of firefly positions (approximate set of solutions) : The object function is non-linear and thus iterative progress of movement of fireflies is indispensable. The movement of each firefly is calculated (Eqn. 4) and applied to the positions in the previous iteration (Eqn. 5). These refined values are taken as approximate for the next iteration and so on.

2.4.1 Terminating *criteria for iterative refinement:* The iterative refinement is continued until convergence criteria are met or user defined maximum number of iterations are completed.

2.5 Tuning of fixed parameters of firefly algorithm : The attraction at zero distance between two fireflies (attractCoef0) and absorption coefficient (absorbCoef) of light in the air are firefly algorithm specific free (tunable) parameters. The randomization reduction coefficient (randReduce) smoothly drives non-deterministic component. The values are chosen empirically based on a priori experience, literature reports (Table 1) etc. Amiri [24] proposed parametric tunings in firefly algorithm using chaotic and self adaptive probabilistic mutation strategies. It enhances the performances of the algorithm. A perusal of literature shows that number of fireflies used varied between 6 to 100 and number of iterations between 25 and 100. The statistical experimental design has a benefit of less number of trials and better end results. Taguchi experimental design which maximizes signal to noise (S/N) ratio is used to choose the parameters of hybrid metaheuristic algorithm comprising of simulated annealing (SA) and firefly [129] components.

Table 1: Literature reports of typical sets free parameters of firefly algorithm

attractCoef0 (β_0)	absorbCoef, (γ)	randReduce α	
1	1	0.2 [0,1]	
1	0.015		
1	1	0.5	
1	1		
0.4	0.5	0.5	
1	0.01 to 0.02 100		Ff19
0.4	0.5		Ff20 Zz1ff
0.4	0.4		Ff20 fp1ff

parameter	Range	stepsize
Nc	[100 to 2000]	100
Np	[10 to 200]	5
β	[0:1 to 0:9]	0.1
γ	[0:1 to 0:9]	0.1
Δm	[0:1 to 0:9]	0.1
V	[10 to 100]	10
T	[1 to 10]	1
NT	[2 to 20]	2

If	Number_of_fireflies >50
Then	CPU time increases

- Absorption coefficient (absorbCoef, γ): The absorption coefficient accounts forvariation of intensity of bioluminescent light. Although, the theoretical range is zero to infinity, in computational approach, it is taken as 1.0. It dictates how fireflies swim in the ocean of optimization landscape and thus is crucial in the speed of convergence. The absorbCoef depends upon distance and sometimes taken as reciprocal of length of design (X) variables (Chart 11). Gandomi et al. [57] used chaotic maps normalized between 0 and 2 for absorbCoef and found a significant improvement in the performance of firefly algorithm. If absorbCoef becomes larger and larger, the light perceived by other fireflies (i.e. attraction) is almost zero (Chart 11b). In other words, the bioluminescence phenomenon has no role in fireflies and tantamount to attraction between them is absent. As such they roam randomly as if it is very thick and foggy surroundings or each firefly is not known to any other firefly.

Chart 11: Calculation of absorbCoef and phenomena at limiting values

Formulae for computation of absorbCoef	
	L : Length of design variable
Theoretical range of absorbCoef : $[0\, to\, \infty]$	
$absorbCoef.h1 = randuniform(0,1)$	
$absorbCoef.h2 = griduniform([0.01\, to\, 100])$	
$absorbCoef.h3 = \frac{iter}{iter\_max} * (absorbCoef\_final - absorbCoef\_initial)$	Input
$mean = 0; var = 0.3;$ $absorbCoef.h4 = \frac{iter}{iter\_max} * randnormal(mean, var) * X(iter)$	

<table>
<tr><td>$absorbCoef.h5 = absorbCoef\ (iter) + \Delta absorbCoef\ (iter)$</td><td rowspan="2">

```
%
%    om_absorbCoef.m   R S Rao 19-10-13
%
function [randu,absorbCoef]=
om_absorbCoef(iter,iterMax,mean,var,
lenghX,absorbCoef_final,absorbCoef_initial)
%
randu = rand(1,1)
absorbCoef.h1 = randu
%
absorbCoef.h2 = griduniform(0.01,100,0.001)
%
absorbCoef.h3 = iter/iterMax *
(absorbCoef_final -absorbCoef_initial)
%
mean =0; var = 1;
absorbCoef.h4 = iter/iterMax *
randn(mean,var)* X(iter)
%
absorbCoef.h5 = absorbCoef+
delta_absorbCoef(iter)
%
absorbCoef.h6 = 1/lengthX
```

output</td></tr>
<tr><td>$absorbCoef.h6 = \frac{1}{lengthX}$</td></tr>
</table>

3 Applications_firefly algorithm: The application of firefly algorithm engulfed diverse disciplines including chemical/ biological/pharmaceutical sciences, medicine, chemical technology, engineering and applied mathematics. This research emerged fast as the translation of the processes (in these disciplines) into models was well nurtured over decades. The corresponding tasks in mathematical parlance are optimization (with single or multiple object functions), parameterization (linear, non-linear), classification/clustering/discrimination (overlapping/non-overlapping), image analysis (pixel, voxel, two color/grey/multi-color), spacio/temporal variation, dimension reduction and orthogonalization/projection. Continuous firefly algorithm has been used in mixed continuous/discrete structural optimization [56] image processing, feature selection, fault detection, antenna design and semantic web composition. Discrete firefly algorithm is used in protein folding [181], simultaneous gene selection from microarray gene expression data [147], and loading pattern optimization of nuclear reactor core of pressured water reactor [126,127], manufacturing cell formation problem [142] and travelling salesman route (TSP) [81]. There are many metrics to test the efficiency of an algorithm. A simple measure is based on number of times a correct solution is arrived (chart 12) compared to number of trials made.

Chart 12: Success rate and quality of solution by an algorithm

If opt_X ≅ (very near to) global optimum

i.e.

$$|opt\_X - global\_optimum| < tol$$

Then run as a successful

$$success\_rate = \frac{trials\_successful}{number\_of\_trials} * 100 \quad \text{Eqn. 7}$$

$$|X^{global} - X^{alg}| \le BU - BL * 10^{-4} \quad \text{Eqn.8}$$

Nall	:	number of all trials
Nsuccessful	:	number of trials which found the solution is successful
Xgb	:	global best by the proposed algorithms
UB and LB	:	upper and lower bounds

BU	:	Upper boundary
BL	:	Lower boundary
X^{global}	:	Global solution known
X^{alg}	:	Solution by the alogirthm

$\lvert . \rvert$	:	Euclidean (or L2) norm

3.1 Chemical science (Economic pollution emission dispatch): Liang and Juarez [184] investigated the know-how of smallest amount of pollution emission with lowest possible cost. This bi- conflicting-objective-goal is approached with normalization of economic dispatch and minimum emission dispatch measures. The results of nature-inspired — firefly/ Virus-optimization / gravitational/ seeker optimisation/ particle swarm/ differential evolution/ harmony search and genetic — algorithms and nonlinear optimisation package TOMLAB in matlab are compared.

Pressurized water reactor (PWR): Poursalehi et al. [126,127] proposed firefly algorithm to obtain a compromised solution of conflicting objectives in fuel loading pattern design of nuclear power reactors of PWR category (Chart 13). The two opposing sub-goals are maximizing Keff (core effective multiplication factor) and maintaining radial PPF (power peaking factor) less than a preset value to comply with safety stipulations with a focus on economics. It results in to extraction of maximum energy and at the same time maintaining fuel integrity.

Chart 13 : Multi-object optimisation fuel loading pattern design PWR nuclear power reactor

$$objFn = W_k * K_{eff} + W_p * (P_{r\max} - PPF) \quad \text{Eqn.9}$$

or

$$objFn = 1.75 * K_{eff} + 2.5 * (1.3 - PPF) \quad \text{Eqn.10}$$

Acronym		Full form	value
Prmax	:	maximum of allowable fuel assembly relative Power	1.3
wk		weighting factor	1.75
wp		weighting factor	2.5

if	Keff increases &
	PPF decreases
Then	objFn increases

if	largest Keff &
	Smallest PPF
Then	objFn is maximum

Chemical equilibria in aqueous solution phase: The chemical equilibria in aqueous solution phase for complex formation of a metal ion with acidic or basic organic ligands include proton-ligand/metal-ligand/protonated or hydroxylated-metal-ligand species [5]. The overall stability constants (parameters with chemical significance) of these interactions (Chart 14) are estimated with experimental probes like glass-electrode / ion-selective-electrode/ UV-Vis spectrophotometer/ ^{1}H-NMR/ ESR using unconstrained non-linear least squares (Marquardt, Gauss-Newton) techniques making use of first and second derivative (gradient [g] Hessian[H]) information to traverse in object function space to estimate $\beta_{m(j),l(j),h(j)H}$ with minimum error in residuals of ingredient concentrations. The data errors are mostly confined to statistical normal distribution and parameters ($\beta_{m(j),l(j),h(j)H}$) also limit stipulations on parametric approach. This is a non-linear mixed integer task with inequality constraints and multiple optimisation criteria. The studies are under progress during the last couple of years [169] to translate the phenomena into distribution free non-linear tasks with constraints using nature inspired algorithms viz. Charged System Search with magnetic forces, honey-bee-mating/honey-bee-foraging, firefly and mosquito hosting modules. The effect of fuzzy/possibility measures and chaos are under consideration instead of statistical Gaussian errors. There is no scope for these features obviously in LETAGROP, SCOGS, Miniquad, superquad, Hyperquad, DALSFEK software packages proposed during second half of last century.

Chart 14: Stability constants of metal complexes in solution

$$m(j)M + l(j)L + h(j)H \xleftrightarrow{\beta_{m(j),l(j),h(j)H}} M_{m(j)}L_{l(j)}H_{h(j)}$$

$$\beta_{m(j),l(j),h(j)H} = \frac{\left[M_{m(j)}L_{l(j)}H_{h(j)}\right]}{FM^{m(j)} * FL^{l(j)} * FH^{h(j)}}$$

Eqn 11

$m(j),\ l(j),\ h(j)H$	:	Coefficients of reactants in mass balance equation
FM, FL, FH	:	Equilibrium (unbounded or free) concentrations of metal ion, ligand and proton
$\beta_{m(j),l(j),h(j)H}$	:	Overall (cumulative) stability constant of complex species $M_{m(j)}L_{l(j)}H_{h(j)}$
ESS		Error sum of squares

$$ESS = \left[\sum_{i=1}^{NP} RM_i^2 + RL_i^2 + RH_i^2\right]$$ Eqn. 12

objFn = ESS ; goal : min(objFn) Eqn. 13

Mixed integer non-linear parameter estimation task

$m(j),\ l(j),\ h(j)H$	:	integer values
$\beta_{m(j),l(j),h(j)H}$	:	floating point
NP	:	Number of experimental data points
RX_i	:	Residual in mass balance equation at i[th] points

Constraints (inequality)

FM, FL, FH	$\geq$	0.0
$\beta_{m(j),l(j),h(j)H}$	$\geq$	0.0
FM, FL, FH	$\leq$	TM, TL, TH

Phase equilibrium/phase stability: Fateen et.al. [54] found that firefly algorithm results in most reliable results in phase and chemical equilibrium calculations of multi-component systems with and without chemical reactions. Phase equilibrium/phase stability computations are pivotal in optimisation of separation processes in chemical engineering. They are mostly multivariable and non-convex with multiple local optima requiring global optimization methods. Earlier, stochastic global optimization algorithms were used for these thermodynamic tasks with a priori knowledge. The output of firefly approach is compared with Covariant Matrix Adaptation-Evolution Strategy (CovMatAdapt-EvolStrat) and Shuffled Complex Evolution (ShufComplxEvol). CovMatAdapt-EvolStrat reaches global minimum with smaller number of iterations.

Continuous casting process: Mauder et al [107] employed firefly algorithm to improve the quality of the steel by continuous casting process. It is pro-influenced by casting speed or cooling rates. With this nature inspired algorithm in optimization process, optimum metallurgical length and core/surface temperatures are achieved.

Thermodynamic model of azeotrope: If compositions of both phases are equal in Ung-Doherty sense, it refers to a thermodynamic condition of coexistence of the two phases under chemical as well as phase equilibrium. This homogeneous reactive azeotrope is modeled by considering physical models of phase/chemical equilibria and corresponding non-linear equations. Earlier, the solution of these systems of equations was arrived at by interval-Newton/generalized bisection algorithms and by hybrid stochastic-deterministic frameworks. Platt et al. [108] made use of Luus-Jaakola adaptive random search and the Firefly algorithm for an industrially important isobutene/methanol/methyl-tert-butyl-ether (MTBE) system with more than one azeotrope. The approach of multiple roots for a nonlinear systems is also applied.

Laser solid freeform fabrication (LSFF) system: Mozaffari et al. [115,116] used firefly algorithm to efficiently predict the architecture of an aggregated-ANN in identifying the behavior of laser solid freeform fabrication (LSFF) system. The results are compared with parallel migrating genetic algorithm (PMGA), fast-SA, differential evolutionary algorithm (DEA), ABC, cuckoo search, differential evolutionary with parent centric crossover (DEPCX), PSO, unified- PSO, shuffle frog leaping algorithm (SFLA) and 'the great salmon run' (TGSR).

3.2 Biochemistry

*Nucleotide data***:** Santander-Jiménez et al. [140] reported discordant genealogical relationships on four real nucleotide data sets applying firefly algorithm satisfying maximum parsimony and maximum likelihood criteria. The results are promising compared to other approaches from the state-of-the-art in Phylogenetics.

DNA, RNA separation with Gel electrophoresis (GE): In Gel electrophoresis (GE), DNA, RNA and protein molecules are separated under electric field applied to a gel matrix. Noor et al. [114] reported image processing techniques for GE image to segment the bands from their background. Multilevel thresholding using Otsu method based on firefly algorithm produced DNA bands and its background.

Protein folding data: Zhang et al. [181] considered 14 sequences of different chain lengths from 18 to 100 of protein folding dataset. A simple strategy is introduced to convert traditional discrete energy function into a continuous one. The simplified energy function totals the distance between all pairs of hydrophobic amino acids. The average outputs of 20 runs of firefly algorithm are better than GA and immune_GA.

Protease production models: A hybrid firefly algorithm is based on swarm- Chemical reaction optimization for synthetic transcriptional oscillators and extracellular protease production models. The results showed that this hybrid method is better than DE, firefly algorithm and chemical reaction optimisation methods. Akaike Information Criterion (AIC) suggested the superiority of the method in choosing a plausible model based on the experimental data. The modified method is of a first rank method in parameter estimation and model selection problems using noisy and incomplete experimental data.

3.3 Medical Diagnostics

Medical image resolution: Du et al. [49] proposed firefly algorithm along with Powell optimisation method for a multi-resolution medical image registration task. The similarity measure used here is the normalized mutual information content. A search for the best value is performed by multi-resolution strategy based on wavelet transformation. Firefly algorithm is employed for the imprecise registration result in the lower resolution image. Then, Powell algorithm is adopted for the higher resolution image to obtain the better registration result. The limitation of mutual information function falling into local optimal values is circumvented with an excellent end result.

Cancer research : Around 28.8 million people were affected with cancer by the year 2008. During this five year period 12.7 million new cases are registered and 7.6 million cancer deaths occurred all over the world. Typical possible sites of cancer development in the human body and prevalent cancer types in different countries are briefed in Chart 15.

Chart 15 (a): Sites of cancer in humans

Lip Oral cavity	Nasopharynx	Pharynx	Oesophagus
Stomach	Liver	Gallbladder	Pancreas
Colorectum	Larynx		
Kidney	Bladder	Brain Nervous system	Thyroid
Hodgkin lymphoma	Non-hodgkin Lymphoma	Multiple Myeloma	Leukaemia
Breast	Cervix uteri		Ovary
Prostate	Testis		
Trachea, bronchus lung	Kaposi sarcoma	Melanoma of skin	Non-melanoma Skin cancers

Chart 15(b): Geographic prevalence of different types of cancers	
Cancer	Prevalent in Geographic area
Breast	global
cervix	Sub-Saharan Africa Southern Asia
prostate	North America, Oceania Northern Western Europe
Stomach	Eastern Asia (including China);
oral cancer	Indian men
Kaposi sarcoma	Sub-Saharan Africa (11 countries)

Diagnosis of breast cancer **:** Breast cancer continues to be dreaded and most prevalent disease in all most all countries exceeding three million patients globally. In computer aided breast cancer diagnosis, classification of benign versus malignant lesions and fibro adenoma from microscopic images of fine needle biopsy samples is a sought after diagnostic tool. Generally, multi-class discrimination techniques are in vogue for the discriminative diagnosis. The cytological material was obtained by fine needle biopsy of breasts of 75 women patients in Regional Hospital in Zielona Góra. Among them, 25 suffer from malignancy and another twenty five with fibroadenoma. Krawczyk and Filipczuk [29,55] used firefly algorithm to discriminate 675 breast lesion images of these patients. Here, each of the classes is represented by an ensemble of one-class classifiers and one-class decomposition strategy is employed. A multi-objective mimetic algorithm is made use in choosing a pool of one-class predictors. These have a characteristic high diversity as well as consistency. The high quality results compared to the state-of-the-art methods endorse the nature inspired algorithm in medical diagnosis.

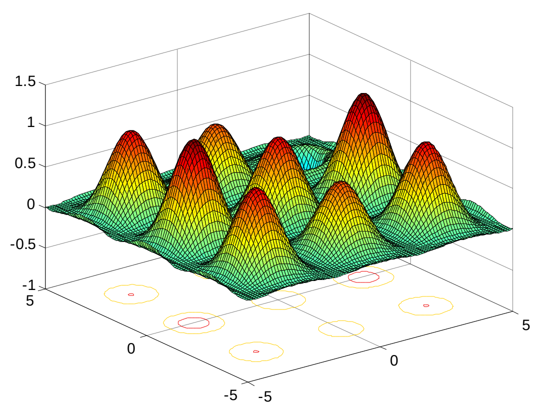
Microarray data**:** The microarray output is high dimensional for each sample and thus classical methods of analysis are inadequate. Srivastava et al. [147] applied two hybrid methods in gene selection and classification of cancerous samples. The first one is a combination of discrete firefly (firefly.Discre) algorithm with SVM and the other is DFA with Random Forests (Rand.Forest) with weighted gene ranking as heuristics. The results on two datasets from Kent Ridge Biomedical Dataset Repository are promising to extract more informative genes in arriving at prediction models with high performance.

3.4 Engineering : Firefly algorithm and its modifications find utility in many pivotal tasks like electric power systems [149,18,125,128,130] economic power load despatch [131] electricity price forecasting [31,102] fuel cell power [121] hydrothermal power [52], wireless sensors [31,152] , PID controllers [83] optical networks [134] mobile/adhoc nets [20,21,109] VLSI routing [27] path optimisation [77,93] design in heating/ventilation/ cooling systems [47], job scheduling [68,173,174], unit commitment [35,37] and flow shop [34,159]. Gorji et al. [59] compared the results of optimizing maximum power output (MPO) and minimum entropy generation (MEG) of an Atkinson cycle. The nature mimicking methods studied are artificial bee colony (ABC) algorithm of Karaboga, improved PSO, firefly computation of Lukasik and self-adaptive penalty function (SAPF)-GA. Mutable smart bee (MSB) algorithm maintains its historical memories for the location and quality of food sources. For this bee, a little chance of mutation during the searching process is also considered. It enhances the ability to mine the data in constrained areas.

Electrical power generation: Chandrasekaran and Simon [35-39] adapted multi-object-optimisation of (MOO) of economic and reliable generation of power by FF. The conflicting objectives viz. fuel cost and

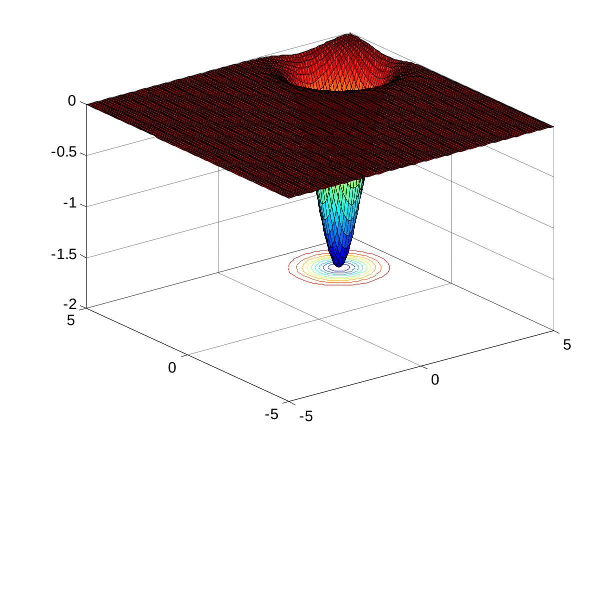
reliability level are reduced to a single objective function using fuzzy weighted optimal deviation. The real coded firefly algorithm tunes the fuzzy membership design variables. The advantage is no need of an expert to set these variables. The algorithm is tested on 100-unit system, IEEE RTS 24-bus system, IEEE 118-bus system and working Taiwan Power (Taipower) 38-unit system over a 24-h period.
Truss structures: Miguel et al. [110,111] employed firefly algorithm for simultaneous optimisation of size, shape and topology of truss structures. The test bed consists of 11-bar, 15-bar-planar, 29-bar-two-tiered, 25-bar-three-tiered truss datasets. Here, the object function is minimisation of structural weight and the constraints are displacement, stress and kinematic stability. It is a mixed integer task, since cross-sectional areas are usually defined by discrete values while other variables are of floating point type. The positive definiteness of the stiffness matrix rules out unstable and singular topologies from possible and mathematically feasible set of solutions. Firefly algorithm provides multiple optima and near-optimal solutions in each run and finally a set of possible set of solutions at the convergence/termination of optimisation process.

3.5 Mathematical tasks : Firefly algorithm is tested for optimisation/classification, clustering [63,179], image compression [72, 154] and image processing [71,157]. It solves stochastic test functions (Xin-she 2010) also, where other algorithms face difficulty. Some of them are Branin, camelback, De Jong, Michalewicz, Shuber, Yang, Shekel, Easom, Goldstein-pric, Griewank, Yang forest, Zaakharo, Styblinski and the typical ones in Table 2. These test functions used for simulated data have different characteristics; discrete/floating point, multimodal/ stochastic /chaotic, discrete/continuous profiles and with breaks/singularity (Figure 1). It is the partial output of om_fnBase.m developed in this laboratory for object oriented database with multiple utilities.

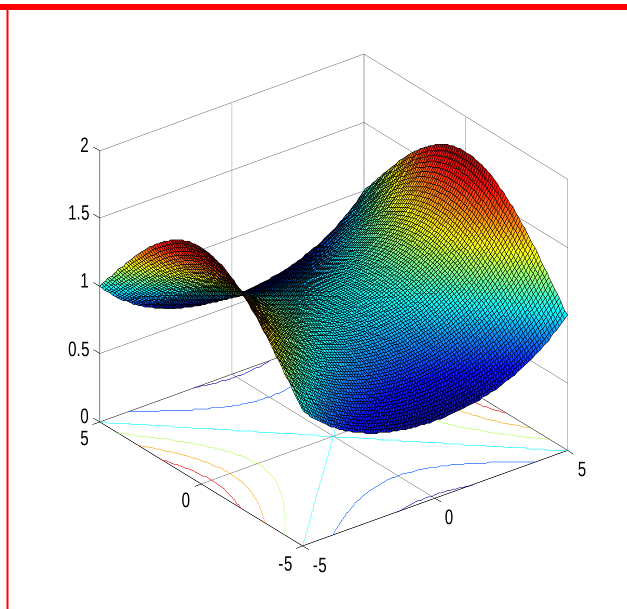



```
fn_Golb_min= [-1]
X_Golb_min = [pi,pi]
```

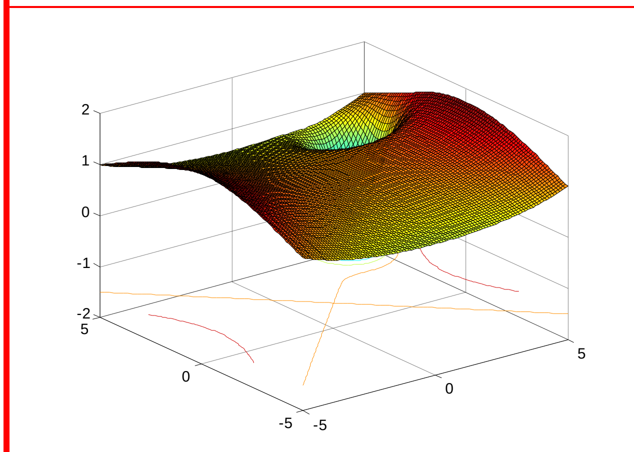
```
%
%   Standing wave
%
c= 5;ll = -c ; ul = c; inc =  .1;
[x1,x2,one] = xygrid(ll,ul,inc);
fnName ='Standing wave'
m = 5; beta=15;
f13 =   exp((-x1/beta).^2*m -
(x2/beta).^2*m ) ;
f14=    -2*exp( - (x1-pi).^2 - (x2-pi).^2
) ;
f11 = f13+f14;
f12     = [(cos(x1)).^2].*[(cos(x2)).^2];
f1 = f11.*f12;
om_xyz_surfcont(x1,x2,f1);
```



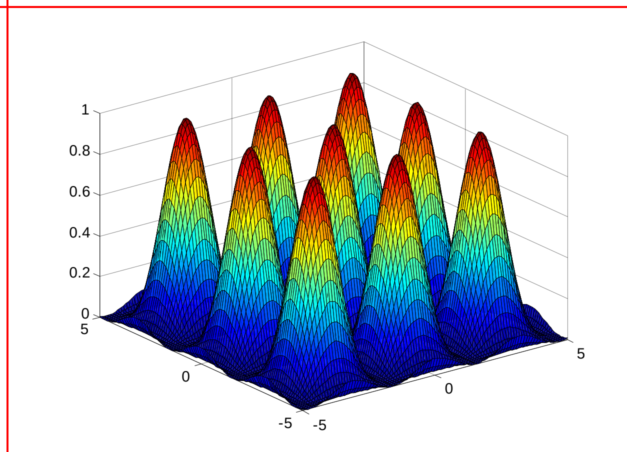
f14= -2*exp(- (x1-pi).^2 - (x2-pi).^2) ;



f13 = exp((-x1/beta).^2*m - (x2/beta).^2*m) ;




```
f11 = f13+f14;
```




```
f12     = [(cos(x1)).^2].*[(cos(x2)).^2];
```

```
f1 = f11.*f12;
```

Fig 1: 3D-surfaces and 2D-isocontours of typical functions used in testing algorithms

Table 2: Mathematical function with their characteristics tested with firefly algorithm

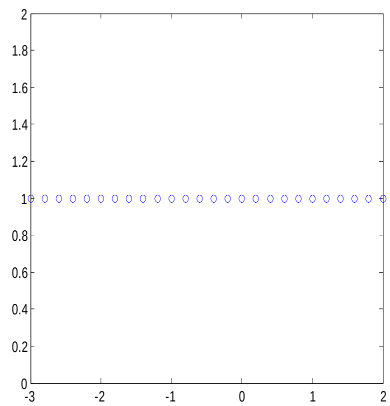
Name_Fn	Characteristics			Name_Fn	Characteristics		
Rosenbrock	Modes [0,1,>1]		unimodal	*Rastrigin*	Modes [0,1,>1]		multimodal
	dimX	:	16		dimX	:	10
	XminGlob	:	[1 1]		rangeX	:	[-5 5]
	FnValue_globMin	:	0	*Ackley*	Modes [0,1,>1]	:	multimodal
	rangeX	:	[-100 100]		dimX	:	128
	16						

Sphere	Modes [0,1,>1]	:	Unimodal		XminGlob	:	0
	dimX	:	10		FnValue_globMin	:	0
	XminGlob	:	[0 0 0]		rangeX	:	[-30,-30]
	FnValue_globMin	:	0	*Griewank*	Modes [0,1,>1]	:	multimodal
	rangeX	:	[-100 100]		dimX	:	16
					XminGlob	:	[0 0 0 .]
					FnValue_globMin	:	0
					rangeX	:	-[600:600]

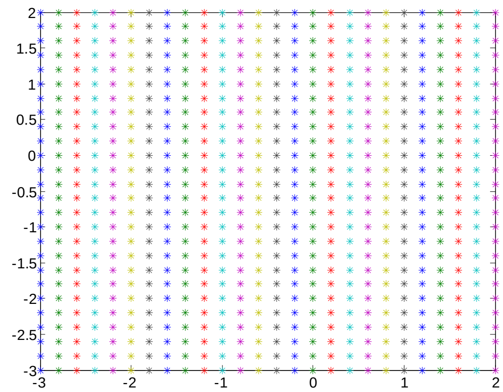
Parameter estimation: Xiong et al. [154] applied firefly algorithm to estimate parameters of multi-output support vector regression. It is employed for accurate interval forecasting of a stock price index task. The prediction of a range of values rather than a point estimate is meaningful in stock price where in profit making investment decisions not only invited, but also at minimum risk of high loss. The results are compared with PSO-MSVR, and GA-MSVR. The same is true for surgery, ICU patient treatment/life-span prediction, synthesis of industrial products, assessment of merit etc. Abu-Mahfouz and Banerjee [78] estimated system parameters of the dynamics of a rotor-stator system with mass imbalance induced rub-impact interactions with firefly algorithm. The results are compared with PSO and differential evolution. Lalithamanohar [88] reported rigorous estimation of parameters of antenna arrays of moderate size using firefly algorithm and invasive weed optimization methods. The firefly algorithm is hybridized with DE and applied to the estimation of parameters of nonlinear biological models. For the test case of arginine catabolism, the results of hybrid methods are far superior to PSO, Nelder-Mead and Firefly algorithms. It also passed through a posteriori practical identifiability test.

Optimisation of training parameters of NN: The parameters of training of neural network models [74,117] have been fine-tuned with firefly algorithm. Senapati et al. [143] employed firefly algorithm to optimise the parameters of local linear wavelet neural network model to classify breast cancer tumor data from University of Wisconsin. Horng et al. [74] applied firefly method for training RBF-NN parameters in classification data sets from UCI repository. Its performance is far better compared to gradient descent (GD), PSO, GA and ABC algorithms. Nandy et al. [117] applied firefly algorithm to back-propagation phase of training of NNs. The simultaneous optimisation of training parameters and number of fireflies are performed in the second phase for dynamic systems. The results are compared with GA-based BP-algorithm.

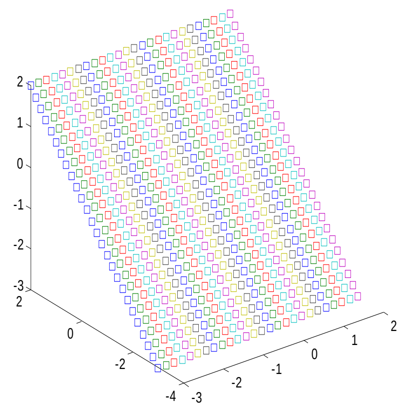
Classification: Horng et al. [72-75] rigorously studied the prospects of combining firefly algorithm with RBF-NN, maximum entropy and vector quantization (VQ) in digital image processing. Linde-Buzo-Gray (LBG) algorithm, a popular one in vector quantization (VQ) gets stuck in local optimal codebook. But, when the results of this VQ are taken as initial solution for firefly algorithm, it results in high quality reconstructed images compared to those from LBG alone or even PSO-LBG. However, the images, of course, are similar with HBMO-LBG output. These authors used maximum entropy based firefly thresholding method for multilevel image selection. The results of the hybrid cooperative-comprehensive learning based PSO algorithm and the HBMA are close with exhaustive search obviously (Fig. 2). Later, they [75] tried minimum cross entropy thresholding (MCET) with firefly algorithm to search for multilevel thresholds for image segmentation. The firefly-MCET method lands in optimum multiple thresholds (confirmed by exhaustive search) efficiently for tasks with less than five thresholds. This study involved PSO, quantum-PSO, HBMO for comparative investigation.



(a)



(b)



(c
)

dim	Range : [-1 :0.01+1]	
1D-	201	
2D-	201 *201	40,401
3D-	201 *201*201	8,120,601
4D-	201 *201*201	1.632e+09
10D-	201 *201*201* 201 *201*201* 201	1.076e+23
100	(201*) …. Hundred terms	2.087e+230

Figure 2: Grid for exhaustive search (a) one D(imensional); (b) two-D; (c) Three-D

Estimation of centroids of clusters: Hassanzadeh and Meybodi [63] employed firefly method to find the centroid of user specified number of clusters. K-means approach is then used to refine cluster parameters.
Inverse problems: One of the objectives of inversion is to define best parameter estimates to arrive at the minimum residual error and in this context the number of parameters is high and frequently the degrees of freedom also very large. They have been transformed into constraint optimization tasks and solved with the heuristic algorithms. Yang [170-172] proposed a uniform unified approach for GA, DE, cuckoo search, PSO and firefly algorithm.

Miscellaneous applications: E-marketing [28], financial portfolio optimization [32], Industrial scale polymerization [43], Wood Berry distillation [43], commerce and meteorology are few other tasks successfully solved with fire fly approach of optimisation. Mardlijah et al. [106] applied firefly algorithm to obtain the best gain scale factor of controller of Fuzzy Type2, which surmounts chattering and is robust towards uncertainties in parameters. Cui and Wang [44] proposed self-organizing time synchronization (STS) method with inspiration from the synchronicity flashes of fireflies. STS is applicable in wireless sensor network for data aggregation and localization.

4. Positive features and Limitations of Firefly algorithm : The features leading to advantages and shortcomings of the algorithm are depicted in chart 16.

Chart16 (a): Advantages of Firefly alg.

- Optima for non-linear, non-convex, non-smooth with a high dimensional search space
- Optimization of multimodal [,165S] functions
- Discovers multiple optima simultaneously (116)
- Excellent global search capability for a SOO goal
- Fireflies aggregate more closely around each optimum
- Solves NP-hard scheduling problems
- Penalty function method can be used to handle the constraints
  - ⇨ Superior to GA and PSO
  - ⇨ GA jumps around multiple optima
- Different fireflies work almost independently
  - ⇨ suitable for parallel implementation
- Superior to ant colony algorithm
- Efficient, Fast computation
- Not a gradient-based search

Chart 16 (b): Limitations of Firefly alg.

- Thorough understanding of metaheuristics with rigorous proof of mathematics is partly a mystery
- Pareto solutions with a uniform distribution is difficult

  Remedy: Heuristic approach

5 Similarity of Firefly alg with other nature mimicking approaches: Firefly algorithm behaves like random search and partially like PSO (Chart 11c). But, with a wise tuning, it smoothly transits not only between these algorithms, but also performs superior, which is not possible by either of them. Ruiz-Vanoye and Díaz-Parra [135] reviewed functional similarities among firefly, GA, Transgenic, ant colony, HBM, ABC, Tabu search, PSO, simulated annealing (SA), artificial immune system (AIS) algorithms. The variation of λ in Marquardt algorithm smoothly moves from steepest descent to first order and second order gradient procedures (GN and NR). Continuous regression emulates multiple linear regression (MLR), principal component regression (PCR) and partial least squares regression (PLSR) algorithms. Modified CR extends mimicking of neural network also.

6. Modifications of firefly algorithm: The modifications in the firefly algorithm are from different perspectives. The first proposed version is improved by automatic splitting of swarms into subgroups (in GSO), use of different (Gaussian, Levy) distributions in random component of displacement vector in binary/floating point modes, employing eliticist choice from GA, global best selection from SA, best, next best from Simplex and scope for Pareto-optimal [24].

Glowworm swarm optimization (GSO): Krishnanand and Ghose [85-87] proposed **GSO** algorithm taking inspiration from varying intensities of light emitted by glowworms with a few more operational knowledge bits cited in chart 17.

Chart 17 :Operational KB of GSO

If	no sufficient_number_of_neighbours limit & no perception_based_on_distance limit	
Then	Glowworms_swarms split into sub-groups	Heuristic-01
If	Sufficient_number_of_ neighbours for a glowworm or range is beyond perception_ range_glowworms	
Then	Effect_of_distant_ glowworms ignored	Heuristic-02

The first heuristic operating in this algorithm (Chart 17) identifies and converges (simultaneously) to multiple peaks of a multi-modal function. The second heuristic promotes automatic splitting of swarms into subgroups leading to movement towards high function value.

Chart 17(b): Glow worm Swarm optimization
Initialisation
γ, β *with chaotic a* lg. Tuning $\frac{\gamma}{\beta} \longleftarrow c_k$
Iteration
Xopt, objFnValue

Binary firefly algorithm: The on and off of a machine (or operation of a power system in unit commitment task) is represented by binary numbers zero and one. Chandrasekharan et al [84] replaced the real values for the positions of fireflies (xi) by binary values (xBi). The refined value of X at an iteration is operated by sigmoid or tanh function (fn) resulting in fn(X). A value is generated from uniform random number (xrandu) ranging between 0 and 1.0. If the value of fn(X) is greater than xrandu, then it is set to 1 and otherwise to zero (table 3).

Table 3: Typical illustration of binary firefly algorithm

Binary	X	Sg(X)	Tanh(X)	xrandu	Sg>xrandu	Tanh>xrandu
1 0	4.1681	0.9848	0.9995	0.2638	1	1
1 1	0.5197	0.6271	0.4775	0.54	1	0

Discrete-Firefly_alg : Poursalehi et al. [126,127] applied discrete firefly algorithm for fuel loading pattern design in Pressurized water reactor. In this method, search space is all possible permutations and proceeds in two phases (chart 18). Hamming distance is used as a measure of discrete distance between two positions of fireflies wherein each firefly corresponds to a permutation. This distance is calculated as the number of non-corresponding elements in the sequence. The change in position for attraction step is calculated and it is followed by that due to non-deterministic (random) component. In the floating point method, attraction step brings the less bright fireflies nearer to brighter ones. The decrease in distance at each iteration step is proportional to the inter distance of the said fireflies in the previous iteration. As Hamming distance is used in discrete firefly algorithm, the permutations should come closer to gather the fireflies towards the optimum. In other words, two permutations get closer as the common elements increase.In the second phase, a random component for the firefly movement is introduced by uniform random number. Integer operation is to confine to discrete values. Jati and Suyanto [81] used discrete distance between two fireflies and discrete movement steps in his discrete version of firefly optimization.

Chart 18: Discrete firefly algorithm

Phase I

Step

1 The common elements of P1and P2 are extracted

P1 =	[4937 6821]
P2 =	[4132 6597]
P1→P2	[4-3- 6---]

2 If permutation P2 attracts permutation P1

Then probability of P1 moving towards P2 [prob(P1→ P2)] is

$$prob\_P1P2 = \exp^{-const*(Hist\_Ham)^2} = \frac{1}{1+const*(Hist\_Ham)^2}$$

3 Generate uniform random number xrandu = randu(1,1)

4
If Prob_P1P2 > xrandu
Then Insert an element from P2 into P1→ P2
Else Retain the element form P1
Endif

5
For All gaps
If The element selected for insertion is already used
Then Skip filling this gap
Endfor

6
If Still empty gaps are present
Then Fill them with unused elements

Step

Phase II

1 Reduced random component for the new position of firefly

2 Convert the position to integer value

3 The permissible values are tested for non-repetition

Modified firefly algorithm: Yuan [175] modified firefly algorithm to solve multi-objective constraint optimization task of rational balance between product family's commonality and difference without transcendental information. The two conflicting objectives are maximization of commonality and minimization of average performance loss. Here, an expression pattern of two-layer artificial firefly structure with common variable ability was tested with optimization design of motor product family.

Mohammadi [183] et al. proposed adaptive tuning of alpha in his adaptive-modified-firefly algorithm. It is applied to assess the uncertainty on the optimal operation management of micro-grids taking into account of the uncertainties of load forecast error, wind turbine (WT) generation, photovoltaic (PV) generation and market price. From the stochastic variables, several processes are generated using PDF (probability density function) and roulette wheel approach. With scenario reduction process, the stochastic task is transformed into a number of deterministic sub-tasks with most dis-similar and probable scenarios.

Firefly algorithm with niching strategy: A niching mechanism was used to select the best compromise solution from the repository such that the population would move towards a smaller search in the space.

Firefly with Min–max approach: Tian et al. [155] introduced inertia weight into firefly algorithm. It promotes global search in the beginning and avoids premature convergence into a local optimum. Further, it carries small inertia weight to local search improving accuracy of optimization. In the standard firefly algorithm, the increase in light intensity and attractiveness sometimes results in repeated oscillations on the position of local/global extrema. The best agents in the succeeding iteration are selected by min–max criterion, ensuring retention of preferences/favors of decision maker in the entire search space. The algorithm now reaches a uniform POF and includes the extreme points of the surface trade-off. It is tested with bi-objective goal with measures the density of the clusters obtained and minimization of external links.

- + Arrives at a set of solutions, corresponding to different trade-offs between the two objectives. Thus, a prior knowledge of a number of communities is not needed
- + Analysis of hierarchy of communities is possible

Fire fly algorithm with guidance matrix: Generation of test cases for software development is complex task and continued to undergo radical change to diminish the failure conditions. Srivatsava et al. [148] used modified fire fly algorithm incorporating guidance matrix in traversing the graph and resulting test paths are optimal critical.

7 Recent advances in firefly algorithm research : The focus is to look for emergence of a stabilized era of computational paradigm, indispensable for progress of mankind with materialistic wealth and physical/mental health. From the conventional Euclidian space, the positions of fireflies have been represented in quaternion space [80] and exponential atmosphere [105]. Binary hybridization with other note worthy nature-inspired algorithms paved way for broadening the scope of fire-fly system. The second component of these hybrid systems are GA [136,64,101], evolutionary strategy (1+1) [156], differential evolution [16], mimetic algorithm, Eagle [171], SA [159], ant colony [58], fuzzy logic [38,39], snake model [48], PSO [82,52] learning automata [19,51] and NNs. Fuzzy ARTMAP by Carpenter and Grossberg school [102] is one of the matured neural network models and coupled with firefly algorithm. Popular mathematical methods viz. FFT [41], wavelet [103], Gaussian distribution function [47], Levy flights [19], K-harmonic means [20,21], minimum/maximum entropy, LBG, chaotic sequence are another set found a place in this pursuit. The classical local search/enhancements procedures, inertia weights [155], random directions and Pareto domains [97] played a role in hybrid systems. It is interesting that genetic operators like mutation, cross over [101] and cellar learning are used instead of coupling the method in toto.

7.1 Quaternion data structure for firefly positions: All nature-mimicking algorithms hitherto developed explore Euclidian space using numerical algebraic operators. Hamilton pointed out that rotations become simpler and unambiguous in (4-D) quaternion space compared to Euclidean space. It has been in wide spread use in controllers of space craft, theoretical physics, protein folding video games. Fister along with Xin-She yang et al. [80] proposed for the first time representation of positions of (virtual/artificial) fireflies in quaternion space (Appendix-3) and obtained astounding results for ten standard optimisation test functions. In quaternion domain, a numerical scalar (zero-order tensor) is represented by 4-dimensional quaternion [169]. The position of a firefly at a moment (or iteration) in d-dimensional space is a d x 4 matrix and for nsol number of fireflies becomes a 3-way data of nsol x ddim x 4 size.

7.1.1 Firefly algorithm in quaternion domain : For every algebraic operation say multiplication, addition and calculation of norm etc., the pertinent rules of quaternion manipulations are used. After convergence, the optimum solution in quaternion space is mapped to floating point domain using the property that 'norm of a quaternion is equal to norm its conjugate'. Chart 19 incorporates pseudo code in quaternion space in column two and firefly in Euclidean space is in column three to appreciate the contrast.

Chart 19: Pseudo-code for **Quaternion firfly alg**

	Floating point	**Quaternion**
Position (X)	Real space	Quaternion space
Initialisation	randu(nsol, ndim)	randuQuat(nsol,ndim,4)

Iterate until convergence \| stopping criteria

operator	**Floating point firfly alg**	**Quaternion firfly alg**
objFnvalue y	y = objFn(X)	yQuat = objFnQuat(Xquat)
sortY along with corresponding X	[Yasc,Xy]=Sort([y,X])	[YascQuat,XyQuat] = Sort([yquat,XQuat])
yBest	Yasc	YascQuat(:,nsol)
Δx	Δx	$\Delta xQuat$
X(:,:,iter)	`Xiter1 = X + delX` `XIter(:,:,iter) = Xiter1`	`XQuatiter1 = XQuat + delXQuat` `XQuatIter(:,:,iter) = XQuatiter1`

endIterate

Optimized solution	[XConv, yConv]	[yquatConv,XQuatConv]
		Mapping quaterion space to floating point (real) spacd
		XConv = Quat2real(XQuatConv)

The advantages of computations in quaternion space (Chart 20) overweigh the increase in memory and using new mode of calculations.

Chart 20: Advantages and limitations of Quaternion-firefly alg

- Apparently more intricate than complex (a + b * i) space
- Four-times more of memory space compared to a floating point scalar

+ Smoother for exploration
+ Search process heads towards more promising areas

Quantum Delta potential well model: Manju and Nigam [105] proposed a quantum Delta potential well model for fireflies, where they are placed in an exponent (surrounding) atmosphere [104]. A global updating operator with weighting function is used. The variation of extinction coefficient with distance between the fireflies is also taken care of.

7.2 Hybridized firefly algorithm

- **Firefly + cellular learning automata**: Hassanzadeh and Meybodi [64] assigned each dimension of the search space of optimization to a one cell of cellular learning automata. A swarm of fireflies are assigned to this cell and they perform the optimization of that dimension. The fringe benefit is the diversity in fireflies' swarm. The results on test cases viz. Sphere, Ackly, Rastrigin, Xin-she yang and Step functions in 10, 20 and 30 dimensional spaces show that the combined function of firefly algorithm and cell learning automata is superior to simple firefly method alone.
- **Firefly + SAA :** The pseudo code for the hybrid firedly and simulated annealing algorithm is described in chart 22 in a nutshell.

Chart 22: Pseudo code of hybrid firefly and SA system

Initialisation
- SA_specific
  - Initial Temp
  - Final Temp
  - Cooling rate
- FA_specific
 	
Termination criteria
- Maxit = 500;
- T <=Tf
	Pseudo code of modules
	- Initialization
	- Parameter tuning
	  - Taguchi orthogonal experimental design
	- FA
	  - Movement_operator
	- SA
	  - Acceptance with probability
	  - Cooling schedule
	- Termination

- Firefly + Chaotic algorithm: Gandomi et al. [57] used 12 different chaotic maps to tune the movement of the fireflies. The chaotic FAs outperform the standard FA in global search ability of multiple optimisation tasks. A chaotic profile is employed to tune the random movement of fireflies. Here, the absorption coefficient was fixed at one. Further, self adaptive probabilistic mutation strategy improves the optimization goal. The size of external memory to store non-dominant solutions was limited by a fuzzy-based clustering technique. From the memory repository, of course best compromised solutions are chosen by a niche mechanism. This promotes reducing search space to smaller and smaller regions in the Pareto-optimal front. Coelho et al. [46] applied hybrid firefly and chaotic algorithms for reliability-redundancy design of an over speed protection system for a gas turbine. The reliability-redundancy is translatable to mixed-integer programming which was solved earlier by dynamic/ integer /mixed-integer nonlinear programming.

Chart 23: 1D- non-invertible chaotic maps		
Chebyshev	Iterative	Sine
Circle	Liebovitch	Singer

Typical chaotic maps studied are indicated in chart 23. A characteristic feature of chaotic system is that small changes in the parameters of chaotic model or the starting values of data lead to widely different behavior of the sequence like stable fixed points, periodic oscillations, non-repetitive sequences, bifurcations and ergodicity.

▶ Gauss/Mouse	▶ Logistic	▶ Sinusoidal
▶ Intermittency	▶ Piecewise	▶ Tent
▶ Tinkerbell map		

The advantages are that it avoids premature convergence. A detailed discussion on chaotic functions in object oriented mode and 3D-surfaces and contour maps will be published separately.

- Firefly + ACO: Xu et al. [161] proposed a hybridization of firefly algorithm with ant colony optimization and applied to group computer animation path design. ACO implies positive feedback mechanisms as well as distributed processing and yields solutions of high accuracy. Thus, this fusion binary hybrid algorithm retains the positive features while at the same time circumventing/nullifying the shortcomings of component methods.

- Firefly + fuzzy ARTMAP: Mandal et al [102] applied firefly algorithm to optimize fuzzy ARTMAP architecture and employed wavelet transform for data filtering. The ill-behaved electricity price time series of Ontario market power system is used for day-ahead forecasting.

- Firefly + PSO: Pengjun et al. [124] incorporated typical aspects of firefly algorithm in PSO. The distances between xi and pbesti and xi and gbest are Cartesian type. The reduced random component of firefly algorithm is added to velocity vector of PSO. It increases the exploration of search space more efficiently. The advantage of this modification resulted in efficient functioning both in continuous and discrete spaces. The algorithm is tested with sphere, Rosenbrock, Rastrigrin and Griewank functions.

- Firefly + Eagle: Yang and Deb [171] performed stochastic optimization with iterative refinement of eagle strategy (comprising of random search with Levy walk) with firefly algorithm.

- Firefly + differential evolution (DiffEvol): Abdullah et al. [16] hybridized DiffEvol with firefly algorithm and estimated parameters in a complex non-linear biological model.

Ternary hybrid systems :

ॐ Firefly + snake model + centripetal force component: The snake model tends to settle in local extrema. Further, it does not converge to the concave edges of object region. The improved snake model includes the centripetal force which has the characteristics of promoting convergence to the concave edges of object region. Du et.al. [48] adapted a two step strategy. The improved Snake model is used for coarse convergence of initial contour of object. This imprecise contour is operated by firefly algorithm (which has good ability of finding the global best optimum) to search the real edges of the object region. This hybrid-nature_ intelligent_algorithm with three modules (viz. improved snake, centripetal and firefly) improves accuracy of image segmentation. State-of-art-of-firefly algorithm in research mode in chart 24 paves way for computational intelligence on a chip.

ॐ

8. Future trends :

Waves in nature inspired algorithms: The inspiration from nature is manifold/ multifold/ omni-fold. Neural networks and genetic-programming were heuristic algorithms based on simple model of brain and genetics proposed during second-half of nineteenth century. Simulated annealing from annealing of glass, genetic algorithm based hereditary characters and tabu-lists from animal behavior followed in the next three decades. ACO and PSO are metaheuristic swarm tools mimicking simple creatures with astounding behavior patterns in foraging, migrating to unknown places for that generation and so on. The first decennium of twenty first century overwhelmed with HBO, firefly, hunting a prey by a herd of wild animals (wolves, tigers), predator-prey, feeding behavior in Aplysia, seeker optimization using humans' intelligent search with their memory/experience/uncertainty-reasoning coupled with exponential number of applications in science and engineering. Physical processes promoted gravitational, charge based magnetic system, artificial physics based on virtual force and water drop optimisation tools during this period. This decade started with Great salmon run, krill-herd, fish swarm, mosquito hosting, Vflight mode of migrating birds, T-cell and teaching_learning, metaheuristics with a positive perspective of aspiring to great knowledge culminating in higher order prospectives. To date, the number of core metaheuristics exceeds twenty five and reported modifications in limelight are around two-hundred and odd. The next wave awaited is for mathematical convergence proofs and in fixing their coordinates on the nature-inspired algorithms virtual map.

Marquardt algorithm and continuous regression are two typical noteworthy instances of hybrid numerical methods amongst many others in the last century. But, rarely, the word 'hybrid algorithm' is loosely used. For example, when different methods are used for different parts of the optimization or data analysis in general, it is just the use of a battery of methods mostly in sequential manner. The procedure in such instances need not be acclaimed as a new hybridisation of two methods.

Future trends in firefly algorithms: The free parameter setting in firefly and other metaheuristic algorithms hitherto are by trial and error approach with a few reports of experimental design. Statistical experimental design coupled with RSM (response surface methodology) and neural networks, successful in calibration, pharmaceutical analysis/preparations, food science etc is worth pursuing tool for optimum values of parameters with less number of trials to depend upon global optima.

Future perspective of software and firmware: Iff (i.e. if and only if) the current-state-of-the-art of science/ technology/ product/ ambitions in multiple-intra-inter-disciplines become a common base of developers/researchers/quality stipulating agencies, then scope increases to realize Self-adaptive-computational-software in the next decade. At the moment, it is a precious baby (paradigms, tools) craving for nutritious feed and (wise) parental (expert) care. The futuristic trend would be amalgamation/ hierarchical/ sequential/ parallel system, with indication of similarities and subtle differences and computations-on-chip.

Intelligence in twenty second century: Already we realized and propagate the concept that yesteryears' knowledge is today's common sense. For example in this decade, we don't raise our eyebrows for atom or even nucleus, MRI, electron, PET etc. Why, because the miracles' of past centuries are now tiny information bits and a subset of knowledge base. But still inquiry and peer probes remain for feeling at home for one molecule in the brain, life on Mars, future (human) habitats on moon and so on. At this pace, the obvious stigma in twenty second century would be 'What is to be called intelligence' and inquisitiveness and curiosity will be focused towards 'meta-intelligence, hyper-intelligence …'. Naturally today's intelligence would just be process knowledge in future. The then hyper intelligence slips down with time to common man's perspective. This is just one side of evolution, if one ignores the impact of mutation and chaos.

Chart 24. State-of-art-of-firefly algorithm in research mode

Type
Floating point
Discrete firefly
binary
Quaternion

Search
Nature inspired
Gradient
Heuristic
Exhaustive

Component models
Deterministic

Deterministic
Grid

Non-deterministic

Polynomial

Non-deterministic Component
Random
Levy Flights
Chaotic
Fuzzy
Possibility
Cloud

Random Component
Rand_uniform(0,1)
rand_normal(mean,sd)
Other distributions
Scaled

MethodBase_ firefly

Modified_ Firefly	Firefly + method
None	**None**
Discrete firecly	FFT
Chaotic_	Wavelet
Alpha-tuning	k-harmonic means
Niching	LBG
Min-max	
Guidance matrix	
Min-max entropy	

Binary Hybrid systems
Firefly +
None
Evolution strategy (1+1)
Differential Evolution
GA
Mimetic
PSO
ACO
Snake
Maximum entropy
Fuzzy
Fuzzy ARTMAP
RBF-NN
VQ
Learning automata

Ternary Hybrid systems
Firefly +
None
snake model + centripetal force

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Appendix-1. Mapping of fireflies movement in nature into Mathematical frame

Distance between fireflies: The distance between two (i,j) fireflies located at the points xi and xj, is Cartesian (Eucledian) distance. Debbarma et al. [139] documents other measures of distances viz. Manhattan or Mahalanobis can be used depending upon the task.

Variation of intensity of light with distance and absorption coefficient: The intensity of light at a specific frequency is inversely proportional to square of the distance between two points (i.e. source and point of observation). The proportionality constant is deemed to intensity of source itself (Eqn.A1.1). Considering the absorption coefficient of medium ($absorCoef$), the intensity of light is

$$I(dist) = e^{-absorCoef * dist^2}$$ (Eqn. A1.2)

Combining these equations leads to Eqn. A1.3 (Table A1.1). At zero distance or when absorbCoef is zero, the intensity of light is same as that of source at all distances. It means that the medium is transparent to that frequency of light. It is like quartz cells are transparent to UV light and glass cells for visible spectrum.

	$I(dist) = \frac{I_{source}}{dist^2}$ Eqn A1.1
If	dist =0
Then	singularity
	Remedy : Exponential form

Table A1.1: KB for intensity of light versus distance

				details
			$I(dist) = I_0 * e^{-absorCoef * dist^2}$ Eqn A1.3	
If	dist =0	Then	$I(dist = 0) = I_0$ Eqn A1.4	$I(dist = 0) = I_0 * e^{-absorCoef * 0^2} = I_0 * e^0 = I_0 *1$
If	absorCoef =0	Then	$I(\text{absorbCoef} = 0) = I_0$	$I(\text{absorbCoef} = 0) = I_0 * e^{-0 * \text{dist}^2} = I_0 * e^0 = I_0 *$
If	absorCoef =1	Then	$I(\text{absorbCoef} = 1) = I_0 * e^{-\text{dist}^2}$ Eqn A1.5	$I(\text{absorbCoef} = 1) = I_0 * e^{-1 * \text{dist}^2}$

Attraction between fireflies: In nature, phenotype/genotype scenarios of living species and environment change continuously. But, theoretical biologists try to reach nature as nearer as possible with mathematical models. The mutual attraction between two fireflies is proportional to light seen by the other fireflies or crudely light reaching them. The proportionality constant is attractCoe and Eqn. A1.2 is a quantitative measure. It is a monotonic decreasing function in distance. If power is two, it reduces to popular inverse law.

$$attract\_fifj \propto light\,seen\,(perceived)\,by\,other\,fireflies$$
$$attract\_fifj = attractCoef * e^{-absorCoef * dist^2}$$ Eqn A1. 6

From the computational jargon of yesteryears promoted calculation of reciprocal $\frac{1}{1 + absorbCoef * dist^2}$ is faster than exponential expressions $e^{-absorCoef * dist^{power}}$. Thus,

$$attract\_fifj = \frac{attractCoef}{1 + absorbCoef * dist^2}$$ Eqn A1. 7.

Eqns. A1.6 and A1.7 are equivalent to third order (Chart A1.1) in dist. O($dist^3$)

Chart A1.1: Series expansion of e^{-x}		
fn(x)		Infinite series expansion $x = absorCoef * dist^2$
e^{-x}	$\cong$	$1 - \frac{1}{1} * x^1 + \frac{1}{2} * x^2 - \frac{1}{3} * x^3 + ...$
$\frac{1}{1+x}$	$\cong$	$1 - x^1 + x^2 - \frac{1}{3} * x^3 + ...$

The general form of eqn. A1.6 is

$$attract\_fifj = attractCoef * e^{-absorCoef * dist^{power}}$$ Eqn. A1.8

and chart A1.2 describes special cases of varying integer powers, attraction/absorption coefficients. It is interesting that attraction between fireflies (attract_fifj) changes in the range of $attractCoef$ **to** $attractCoef * e^{-1}$ and attraction coefficient varies between $attractCoef$ **and** $\frac{attractCoef}{2}$.

Chart A1.2: Numerical expert system for attraction between two fireflies			
$attract\_fifj = attractCoef * e^{-absorCoef * dist^{power}}$ Eqn A1.8			
			Eqn A1.8 reduces to
If	Power = 2	Then	Inverse square law Eqn A1.1
If	Power = 0	Then	$attractCoef * e^{-absorCoef} = constant$
If	attractCoef = 0	Then	0
If	attractCoef = 1& absorbCoef = 0	Then	1
If	absorbCoef = 0 & attractCoef = 1	Then	$attractCoef = constant$

Chart **A1.2**(b):Knowledge bits for numerical expert system of absorbCoef
$attract\_xixj = attractCoef0 * \exp^{-absorbCoef*(dist)^{power}}$
$\Delta x(i,j,iter) = attract\_xixj * [x(i,j,iter) - x(j,j,iter)] + randReduce * rand$
$x(i,j,iter+1) = x(j,j,iter) + \Delta x(i,j,iter)$
If $absorbCoef \to 0$

Then $attract - fifj \rightarrow attractCoef\_0$

If $absorbCoef\ is\ fine\_tuned$

$\text{in the } range\ [0 < absorbCoef < large]$

Then Firefly alg emulates PSO, random search and goes beyond

\+ Firefly alg outperforms both PSO and random search

If $absorbCoef > large\ [\text{or } absorbCoef \rightarrow \propto]$

Then attractiveness decreases dramatically

If Brightness decreases

Then all fireflies are short-sighted or equivalently fly in a thick foggy sky

If attractiveness decreases

Then all fireflies move almost randomly

If $attractCoef\_0 = 0$

Then Attract_f1f2 = 0

If $attract\_0 = 1$

Then the brightest firefly strongly determines the other fireflies' position [cooperative local search in neighborhood]

Chart **A1.2(** c) : Emulation of other nature inspired algorithms at limiting values absorbCoef

If $absorbCoef \rightarrow 0$

Then Firefly emulates PSO

If $absorbCoef >$ large

Then Firefly alg reduces to random search

Appendix-2: Chemistry and biochemistry of firefly *luminescence* : In the beetle order Coleoptera, Lampyridae is a family of insects having wings. The colloquial term is fireflies or lightning bugs for their conspicuous bioluminescence (Fig A1-1).

(a) Firefly with greenish-yellowish light

(b) Firefly with yellowish light

(c) Firefly with white light

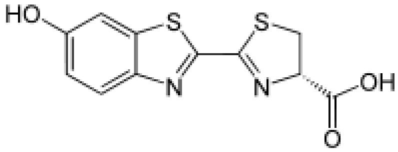
(d) Fireflies in the woods (Credit : wikipedia)

(d) bioluminescent algae below water
cosmic light above the water
credit: *Phil Hart*

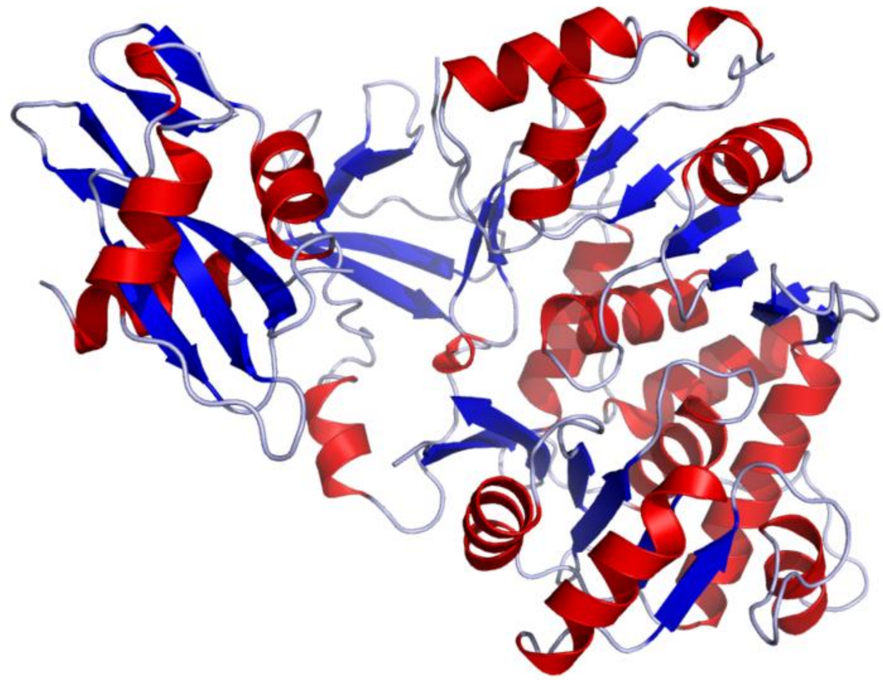
(e) Firefly squid

Fig A1-1: Fireflies , algae and firefly squid with different luminescent characteristics

Luciferin in presence oxygen and ATP undergoes chemical reactions (CR) producing electronically excited oxyluciferin and AMP [CR. 1 to 3]. When it returns to ground state, a photon of light is emitted. The bioluminescence, heatless, bluish-green light, of fireflies is due to catalytic activity of luciferase and ATP on oxidation of the luciferin, a chemical present in the cells. The chemical structures of luciferin and enzyme luciferase are in figure A1-2. Luciferyl adenylate can additionally participate in a side reaction with O_2 to form hydrogen peroxide and dehydroluciferyl-AMP.



(4*S*)-2-(6-hydroxy-1,3-benzothiazol-2-yl)-4,5-dihydrothiazole-4-carboxylic acid



luciferin + ATP → luciferyl adenylate + PPi	**CR. 01**
luciferyl adenylate + O2 → oxyluciferin(excited)+ AMP	**CR. 02**
oxyluciferin(excited) → oxyluciferin(ground state) + light -ok	**CR. 03**

Fig A1-2: Bioluminiscence in fireflies

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